

**MOORESTOWN TOWNSHIP PUBLIC SCHOOLS
MOORESTOWN, NEW JERSEY**

**Moorestown High School
Mathematics**

**Multivariable Calculus & Differential Equations
*Grade 12***

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[Course Description and Fundamental Concepts](#)

This course explores topics normally covered in two semesters of college. Major topics in Multivariable Calculus include: vectors and the geometry of space, vector-valued functions, functions of several variables, multiple integration, and vector analysis. Major topics in Differential Equations include: first-order and higher order differential equations & modeling, series solutions of linear equations, The Laplace Transform, systems of linear first-order differential equations and numerical solutions of ordinary differential equations. This course is designed for students who have completed the AP Calculus BC course and desire to do additional college-level work. For both classes, students are expected to enter this course with a completion of Calculus BC or equivalent. Students will also be required to utilize the software Octave or MATLAB. There will be projects assigned associated with the techniques learned throughout each semester.

New Jersey Student Learning Standards (NJSLS)

Subject/Content Standards

Include grade appropriate subject/content standards that will be addressed

N-RN	<i>The Real Number System</i>
	<p>A. Extend the properties of exponents to rational exponents.</p> <ol style="list-style-type: none"> 1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5. 2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.
	<p>B. Use properties of rational and irrational numbers.</p> <ol style="list-style-type: none"> 3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
N-Q	<i>Quantities</i>
	<p>A. Reason quantitatively and use units to solve problems.</p> <ol style="list-style-type: none"> 1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. 2. Define appropriate quantities for the purpose of descriptive modeling. 3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
N-CN	<i>The Complex Number System</i>
	<p>A. Perform arithmetic operations with complex numbers.</p> <ol style="list-style-type: none"> 1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. 2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. 3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
	<p>B. Represent complex numbers and their operations on the complex plane.</p>

	<p>4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.</p> <p>5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}i)$ has modulus 2 and argument 120°.</p> <p>6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.</p>
	<p>C. Use complex numbers in polynomial identities and equations.</p> <p>7. Solve quadratic equations with real coefficients that have complex solutions.</p> <p>8. (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$. New Jersey Student Learning Standards for Mathematics 6</p> <p>9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.</p>
N-VM	Vector and Matrix Quantities
	<p>A. Represent and model with vector quantities.</p> <p>1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, v, $\ v\$, v).</p> <p>2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.</p> <p>3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.</p>
	<p>B. Perform operations on vectors.</p> <p>4. (+) Add and subtract vectors.</p> <p>a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.</p> <p>b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.</p> <p>c. Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.</p> <p>5. (+) Multiply a vector by a scalar.</p>

	<p>a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.</p> <p>b. Compute the magnitude of a scalar multiple cv using $\ cv\ = c v$. Compute the direction of cv knowing that when $c v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$).</p>
	<p>C. Perform operations on matrices and use matrices in applications.</p> <p>6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.</p> <p>7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.</p> <p>8. (+) Add, subtract, and multiply matrices of appropriate dimensions.</p> <p>9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.</p> <p>10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.</p> <p>11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.</p> <p>12. (+) Work with 2×2 matrices as a transformation of the plane, and interpret the absolute value of the determinant in terms of area.</p>
A-SSE	Seeing Structure in Expressions
	<p>A. Interpret the structure of expressions</p> <p>1. Interpret expressions that represent a quantity in terms of its context.¹</p> <p>a. Interpret parts of an expression, such as terms, factors, and coefficients.</p> <p>b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P</p> <p>2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.</p>
	<p>B. Write expressions in equivalent forms to solve problems</p> <p>3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★</p> <p>a. Factor a quadratic expression to reveal the zeros of the function it defines.</p>

	<p>b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.</p> <p>c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.</p> <p>4. Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.</p>
A-APR	Arithmetic with Polynomials and Rational Expressions
	<p>A. Perform arithmetic operations on polynomials</p> <p>1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.</p>
	<p>B. Understand the relationship between zeros and factors of polynomials</p> <p>2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.</p> <p>3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.</p>
	<p>C. Use polynomial identities to solve problems</p> <p>4. Prove polynomial identities and use them to describe numerical relationships. For example, the difference of two squares; the sum and difference of two cubes; the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.</p> <p>5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.</p>
	<p>D. Rewrite rational expressions</p> <p>6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.</p> <p>7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.</p>
A-CED	Creating Equations ★
	A. Create equations that describe numbers or relationships

	<ol style="list-style-type: none"> 1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. 3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. 4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R.
A-REI	Reasoning with Equations and Inequalities
	<p>A. Understand solving equations as a process of reasoning and explain the reasoning</p> <ol style="list-style-type: none"> 1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. 2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
	<p>B. Solve equations and inequalities in one variable</p> <ol style="list-style-type: none"> 3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. 4. Solve quadratic equations in one variable. <ol style="list-style-type: none"> a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
	<p>C. Solve systems of equations</p> <ol style="list-style-type: none"> 5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. 6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. 7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$. 8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.

	<p>9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).</p>
	<p>D. Represent and solve equations and inequalities graphically</p> <p>10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).</p> <p>11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★</p> <p>12. Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.</p>
F-IF	Interpreting Functions
	<p>A. Understand the concept of a function and use function notation</p> <p>1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.</p> <p>2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.</p> <p>3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.</p>
	<p>B. Interpret functions that arise in applications in terms of the context</p> <p>4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★</p> <p>5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. ★</p> <p>6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★</p>

	<p>C. Analyze functions using different representations</p> <p>7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★</p> <p>a. Graph linear and quadratic functions and show intercepts, maxima, and minima.</p> <p>b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</p> <p>c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.</p> <p>d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.</p> <p>e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</p> <p>8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</p> <p>a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.</p> <p>b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.</p> <p>9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</p>
F-BF	Building Functions
	<p>A. Build a function that models a relationship between two quantities</p> <p>1. Write a function that describes a relationship between two quantities. ★</p> <p>a. Determine an explicit expression, a recursive process, or steps for calculation from a context.</p> <p>b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</p> <p>c. (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.</p> <p>2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ★</p>
	B. Build new functions from existing functions

	<p>3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.</p> <p>4. Find inverse functions.</p> <p>a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.</p> <p>b. (+) Verify by composition that one function is the inverse of another.</p> <p>c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.</p> <p>5. (+) Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.</p>
F-LE	Linear and Exponential Models ★
	<p>A. Construct and compare linear and exponential models and solve problems</p> <p>1. Distinguish between situations that can be modeled with linear functions and with exponential functions.</p> <p>a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.</p> <p>b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.</p> <p>c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.</p> <p>2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).</p> <p>3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.</p> <p>4. Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.</p>
	<p>B. Interpret expressions for functions in terms of the situation they model</p> <p>5. Interpret the parameters in a linear or exponential function in terms of a context.</p>
F-TF	Trigonometric Functions
	<p>A. Extend the domain of trigonometric functions using the unit circle</p> <p>1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.</p>

	<p>2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.</p> <p>3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for πx, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.</p> <p>4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.</p>
	<p>B. Model periodic phenomena with trigonometric functions</p> <p>5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★</p> <p>6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.</p> <p>7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. ★</p>
	<p>C. Prove and apply trigonometric identities</p> <p>8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.</p> <p>9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.</p>
S-ID	Interpreting Categorical and Quantitative Data
	<p>A. Summarize, represent, and interpret data on a single count or measurement variable</p> <p>1. Represent data with plots on the real number line (dot plots, histograms, and box plots).</p> <p>2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.</p> <p>3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).</p> <p>4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.</p>
	<p>B. Summarize, represent, and interpret data on two categorical and quantitative variables</p>

	<p>5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.</p> <p>6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.</p> <p>a. Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.</p> <p>b. Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology.</p> <p>c. Fit a linear function for a scatter plot that suggests a linear association.</p>
	<p>C. Interpret linear models</p> <p>7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.</p> <p>8. Compute (using technology) and interpret the correlation coefficient of a linear fit.</p> <p>9. Distinguish between correlation and causation.</p>

21st-Century Skills and Technology Integration ([Standard 8](#))

List appropriate units below for which strands (A through F) will be addressed

Standard 8.1 (K-12)		Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.
Unit Addressed	Strand Letter	Standard Description
Units 1, 2, 3, 4, 5, 6, 7, 9, 10	Strand A	Technology Operations and Concepts: Students demonstrate a sound understanding of technology concepts, systems, and operations.
Units 1, 2, 3, 4, 5, 6, 7, 9, 10	Strand B	Creativity and Innovation: Students demonstrate creative thinking, construct knowledge and develop innovative products and process using technology.
Units 1, 2, 3, 4, 5, 6, 7, 9, 10	Strand C	Communication and Collaboration: Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
Units 1, 2, 3, 4, 5, 6, 7, 9, 10	Strand D	Digital Citizenship: Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.

Units 1, 2, 3, 4, 5, 6, 7, 9, 10	Strand E	Research and Information Fluency: <i>Students apply digital tools to gather, evaluate, and use information.</i>
Units 1, 2, 3, 4, 5, 6, 7, 8, 9, 10	Strand F	Critical thinking, problem-solving, and decision making: <i>Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.</i>

Career Ready Practices ([Standard 9](#))

List appropriate units below for which CRPs will be addressed

Unit Addressed	Standard #	Standard Description
	CRP1	<i>Act as a responsible and contributing citizen and employee.</i>
Units 1, 2, 3, 4, 5, 6, 7, 8, 9, 10	CRP2	<i>Apply appropriate academic and technical skills.</i>
	CRP3	<i>Attend to personal health and financial well-being.</i>
	CRP4	<i>Communicate clearly and effectively and with reason.</i>
Units 6, 7, 9, 10	CRP5	<i>Consider the environmental, social and economic impacts of decisions.</i>
Units 1, 2, 3, 4, 5, 6, 7, 8, 9, 10	CRP6	<i>Demonstrate creativity and innovation.</i>
Units 1, 2, 3, 4, 5, 6, 7, 9, 10	CRP7	<i>Employ valid and reliable research strategies.</i>
Units 1, 2, 3, 4, 5, 6, 7, 8, 9, 10	CRP8	<i>Utilize critical thinking to make sense of problems and persevere in solving them.</i>
Units 1, 2, 3, 4, 5, 6, 7, 8, 9, 10	CRP9	<i>Model integrity, ethical leadership, and effective management.</i>
	CRP10	<i>Plan education and career paths aligned to personal goals.</i>
Units 1, 2, 3, 4, 5, 6, 7, 8, 9, 10	CRP11	<i>Use technology to enhance productivity.</i>
	CRP12	<i>Work productively in teams while using cultural global competence</i>

Interdisciplinary Connections

List any other content standards addressed as well as appropriate units

Visual & Performing Arts Integration ([Standard 1](#))

List appropriate units below for which standards (1.1 through 1.4) may be addressed

Unit Addressed	Standard #	Standard Description
	Standard 1.1	The Creative Process: All students will demonstrate an understanding of the elements and principles that govern the creation of works of art in dance, music, theatre, and/or visual art.
	Standard 1.2	History of the Arts and Culture: All students will understand the role, development, and influence of the arts throughout history and across cultures.
	Standard 1.3	Performing/Presenting/Producing: All students will synthesize those skills, media, methods, and technologies appropriate to creating, performing, and/or presenting works of art in dance, music, theatre, and/or visual art.
	Standard 1.4	Aesthetic Responses & Critique Methodologies: All students will demonstrate and apply an understanding of arts philosophies, judgment, and analysis to works of art in dance, music, theatre, and/or visual art.

Other Interdisciplinary Content Standards

List appropriate units below for any other content/standards that may be addressed

Unit Addressed	Content / Standard #	Standard Description
Units 6, 7, 9, 10	WHST.9-12.2	Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes.
Units 1, 2, 3, 4, 5, 6, 7, 9, 10	WHST.9-12.7	Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.
Units 1, 2, 3, 4, 5, 6, 7, 8, 9, 10	WHST.11-12.8	Gather relevant information from multiple authoritative print and digital sources, using advanced searches effectively; assess the strengths and limitations of each source in terms of the

		specific task, purpose, and audience; integrate information into the text selectively to maintain the flow of ideas, avoiding plagiarism and overreliance on any one source and following a standard format for citation.
Units 1, 2, 3, 4, 5, 6, 7, 8, 9, 10	WHST.9-12.9	Draw evidence from informational texts to support analysis, reflection, and research.
Units 6, 7, 8, 9, 10	HS-LS1-7	Use a model to illustrate that cellular respiration is a chemical process whereby the bonds of food molecules and oxygen molecules are broken and the bonds in new compounds are formed resulting in a net transfer of energy.
Units 1, 2, 3, 4, 5, 6, 7, 8, 9, 10	HS-ETS1-4	Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.
Units 6, 7, 9, 10	HS-PS1-3	Plan and conduct an investigation to gather evidence to compare the structure of substances at the bulk scale to infer the strength of electrical forces between particles.
Units 6, 7, 9, 10	HS-PS1-4	Develop a model to illustrate that the release or absorption of energy from a chemical reaction system depends upon the changes in total bond energy.
Units 6, 7, 9, 10	HS-PS1-5	Apply scientific principles and evidence to provide an explanation about the effects of changing the temperature or concentration of the reacting particles on the rate at which a reaction occurs.
Units 6, 7, 9, 10	HS-PS1-6	Refine the design of a chemical system by specifying a change in conditions that would produce increased amounts of products at equilibrium.
Units 6, 7, 9, 10	HS-PS1-8	Develop models to illustrate the changes in the composition of the nucleus of the atom and the energy released during the processes of fission, fusion, and radioactive decay.

Pacing Guide (All Dates are approximate based on the school calendar)

Unit/ Topic	Month (w/Approx number of Teaching Days)
UNIT 1 Vectors - Chapter 10	September (~19 days)
UNIT 2 Vector functions - Chapter 11	October (~19 days)
UNIT 3 Multivariable functions - Chapter 12	November (~16 days)
UNIT 4 Double and triple integrals - Chapter 13	December (~15 days)
UNIT 5 Vector analysis - Chapter 14	January (~18 days)
UNIT 6 First-order equations - Chapter 1	February (~18 days)
UNIT 7 Second-order linear equations - Chapter 2	March (~15-20 days)
UNIT 8 First and second-order linear equations using power series - Supplemental	April (~15-20 days)
UNIT 9 Laplace transforms - Chapter 3	May (~18 days)
UNIT 10 Linear systems - Chapter 4	June (~15 days)

[Units](#)

Contact the Content Supervisor for unit details.