MOORESTOWN TOWNSHIP PUBLIC SCHOOLS MOORESTOWN, NEW JERSEY

Moorestown High School Mathematics

Honors PreCalculus *Grades 10-11*

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Contents

Administration

Course Description and Fundamental Concepts

New Jersey Student Learning Standards

Pacing Guide

Units

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Course Description and Fundamental Concepts

This rigorous course requires students to use independent thinking. Reading and problem solving are emphasized throughout the course. Major concepts include: trigonometry, circular functions, logarithms, exponential functions, complex numbers, polar coordinates and equations, mathematical induction, sequences and series and their limits, logic, polynomials, rational functions, counting techniques, probability and limits & differentiation. The range of concepts is much greater than that of the college prep course, the pace is faster, and students are expected to share responsibility for their learning. A graphing calculator, which is an integral part of the curriculum, is required.

New Jersey Student Learning Standards (NJSLS)

Subject/Content Standards

Include grade appropriate subject/content standards that will be addressed

N-CN The Complex Number System

- B. Represent complex numbers and their operations on the complex plane.
 - 4. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
 - 5. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \text{sqrt}(3i))^3 = 8$ because (-1 + sqrt(3i)) has modulus 2 and argument 120°.
 - 6. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
- C. Use complex numbers in polynomial identities and equations.
 - 7. Solve quadratic equations with real coefficients that have complex solutions.
 - 8. Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as (x + 2i)(x 2i).
 - 9. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

N-VM Vector and Matrix Quantities

- A. Represent and model with vector quantities.
 - 1. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v).
 - 2. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
 - 3. Solve problems involving velocity and other quantities that can be represented by vectors.
- B. Perform operations on vectors.
 - 4. Add and subtract vectors.
 - a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
 - b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
 - c. Understand vector subtraction v w as v + (-w), where -w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
 - 5. Multiply a vector by a scalar.
 - a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(vx, vy) = (cvx, cvy).
 - b. Compute the magnitude of a scalar multiple cv using ||cv|| = |c|v. Compute the direction of cv knowing that when $|c|v \neq 0$, the direction of cv is either along v (for c > 0) or against v (for c < 0).

- C. Perform operations on matrices and use matrices in applications.
 - 6. Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
 - 7. Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
 - 8. Add, subtract, and multiply matrices of appropriate dimensions.
 - 9. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
 - 10. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
 - 11. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
 - 12. Work with 2×2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area.

A-SSE Seeing Structure in Expressions

- A. Interpret the structure of expressions
 - 1. Interpret expressions that represent a quantity in terms of its context.
 - a. Interpret parts of an expression, such as terms, factors, and coefficients.
 - b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)ⁿ as the product of P and a factor not depending on P
 - 2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 y^4$ as $(x^2)^2 (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 y^2)(x^2 + y^2)$.
- B. Write expressions in equivalent forms to solve problems
 - 3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
 - a. Factor a quadratic expression to reveal the zeros of the function it defines.
 - b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
 - c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.01212t$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.
 - 4. Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.

A-APR Arithmetic with Polynomials and Rational Expressions

- A. Perform arithmetic operations on polynomials
 - 1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- B. Understand the relationship between zeros and factors of polynomials
 - 2. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x a is p(a), so p(a) = 0 if and only if (x a) is a factor of p(x).
 - 3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

- C. Use polynomial identities to solve problems
 - 4. Prove polynomial identities and use them to describe numerical relationships. For example, the difference of two squares; the sum and difference of two cubes; the polynomial identity $(x^2 + y^2)^2 = (x^2 y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.
 - 5. Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
- D. Rewrite rational expressions
 - 6. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
 - 7. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

A-REI Reasoning with Equations and Inequalities

- C. Solve systems of equations
 - 5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
 - 6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
 - 7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = -3x and the circle $x^2 + y^2 = 3$.
 - 8. Represent a system of linear equations as a single matrix equation in a vector variable.
 - 9. Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).
- D. Represent and solve equations and inequalities graphically
 - 10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
 - 11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
 - 12. Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

F-IF Interpreting Functions

- A. Understand the concept of a function and use function notation
 - 1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
 - 2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

- 3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1).
- B. Interpret functions that arise in applications in terms of the context
 - 4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
 - 5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
 - 6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- C. Analyze functions using different representations
 - 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
 - a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
 - b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
 - c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
 - d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
 - e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
 - 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
 - a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
 - b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.
 - 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

F-BF Building Functions

- A. Build a function that models a relationship between two quantities
 - 1. Write a function that describes a relationship between two quantities.
 - a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
 - b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

- c. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.
- B. Build new functions from existing functions
 - 3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
 - 4. Find inverse functions.
 - a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2 \times 3$ or f(x) = (x+1)/(x-1) for $x \ne 1$.
 - b. Verify by composition that one function is the inverse of another.
 - c. Read values of an inverse function from a graph or a table, given that the function has an inverse
 - d. Produce an invertible function from a non-invertible function by restricting the domain.
 - 5. Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.

F-LE Linear and Exponential Models

- A. Construct and compare linear and exponential models and solve problems
 - 1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
 - a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
 - b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
 - c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
 - 2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
 - 3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
 - 4. Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to ab^{ct} = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
- B. Interpret expressions for functions in terms of the situation they model
 - 5. Interpret the parameters in a linear or exponential function in terms of a context.

F-TF Trigonometric Functions

- A. Extend the domain of trigonometric functions using the unit circle
 - 1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

- 2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- 3. Use special triangles to determine geometrically the values of sine, cosine, tangent for pi/3, pi /4 and pi /6, and use the unit circle to express the values of sine, cosines, and tangent for pi x, pi +x, and 2 pi -x in terms of their values for x, where x is any real number.
- 4. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
- B. Model periodic phenomena with trigonometric functions
 - 5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
 - 6. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
 - 7. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
- C. Prove and apply trigonometric identities
 - 8. Prove the Pythagorean identity $\sin 2(\theta) + \cos 2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
 - 9. Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

G-SRT Similarity, Right Triangles, and Trigonometry

- D. Apply trigonometry to general triangles
 - 9. Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
 - 10. Prove the Laws of Sines and Cosines and use them to solve problems.
 - 11. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Mathematical Practice Standards

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

21st-Century Skills and Technology Integration (Standard 8)

List appropriate units below for which strands (A through F) will be addressed

Standard 8.1 (K-12)		Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.
Unit Addressed	Strand Letter	Standard Description
Units 1, 2, 3, 4, 5	Strand A	Technology Operations and Concepts: Students demonstrate a sound understanding of technology concepts, systems, and operations.
Units 1, 2, 3, 4, 5	Strand B	Creativity and Innovation: Students demonstrate creative thinking, construct knowledge and develop innovative products and process using technology.
Units 1, 2, 3, 4, 5	Strand C	Communication and Collaboration: Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
	Strand D	Digital Citizenship: Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.
Units 1, 2, 3, 4, 5	Strand E	Research and Information Fluency: Students apply digital tools to gather, evaluate, and use information.
Units 1, 2, 3, 4, 5	Strand F	Critical thinking, problem-solving, and decision making: Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.

Career Ready Practices (Standard 9)

List appropriate units below for which CRPs will be addressed

Unit Addressed	Standard #	Standard Description	
	CRP1	Act as a responsible and contributing citizen and employee.	
Units 1, 2, 3, 4, 5	CRP2	Apply appropriate academic and technical skills.	

	CRP3	Attend to personal health and financial well-being.	
Units 1, 2, 3, 4, 5	CRP4	Communicate clearly and effectively and with reason.	
	CRP5	Consider the environmental, social and economic impacts of decisions.	
Units 1, 2, 3, 4, 5	CRP6	Demonstrate creativity and innovation.	
	CRP7	Employ valid and reliable research strategies.	
Units 1, 2, 3, 4, 5	CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.	
	CRP9	Model integrity, ethical leadership, and effective management.	
	CRP10	Plan education and career paths aligned to personal goals.	
Units 1, 2, 3, 4, 5	CRP11	Use technology to enhance productivity.	
Units 1, 2, 3, 4, 5	CRP12	Work productively in teams while using cultural global competence	

Interdisciplinary Connections

List any other content standards addressed as well as appropriate units

Visual & Performing Arts Integration (Standard 1)

List appropriate units below for which standards (1.1 through 1.4) <u>may be addressed</u>

Unit Addressed	Standard #	Standard Description	
Standard 1.1		The Creative Process: All students will demonstrate an understanding of the elements and principles that govern the creation of works of art in dance, music, theatre, and/or visual art.	
	Standard 1.2	History of the Arts and Culture: All students will understand the role, development, and influence of the arts throughout history and across cultures.	
	Standard 1.3	Performing/Presenting/Producing: All students will synthesize those skills, media, methods, and technologies appropriate to creating, performing, and/or presenting works of art in dance, music, theatre, and/or visual art.	
	Standard 1.4	Aesthetic Responses & Critique Methodologies: All students will demonstrate and apply an understanding of arts philosophies, judgment, and analysis to works of art in dance, music, theatre, and/or visual art.	

Other Interdisciplinary Content Standards
List appropriate units below for any other content/standards that <u>may be addressed</u>

Unit Addressed	Content / Standard #	Standard Description
Units 1, 2, 5	NJSLSA.R7	Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
Units 1, 4	9.1.12.A.3	Analyze the relationship between various careers and personal earning goals.
Units 1, 5	HS-ETS1-2	Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.
Unit 1	9.1.12.A.9	Describe and calculate interest and fees that are applied to various forms of spending, debt, and saving.
Unit 1	9.1.12.B.2	Develop a model based on evidence of Earth's interior to describe the cycling of matter by thermal convection.
Unit 2	MS-PS4-1	Use mathematical representations to describe a simple model for waves
Units 2, 5	NJSLSA.W1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
Units 1, 2, 3, 4, 5	NJSLSA.SL4	Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
Units 3, 5	HS-ETS1-2	Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

Pacing Guide (All Dates are approximate based on the school calendar)

Unit/ Topic	Month (w/Approx number of Teaching Days)
UNIT 1: Functions Linear, Polynomial Functions	September (~19 days)
UNIT 1: Functions Rational, Exponential, Logarithmic Functions	October (~19 days)
UNIT 2: Trigonometry Unit Circle, Right Triangle Trig, Graphing, Inverses, Applications	November (~16 days)
UNIT 2: Trigonometry Analytic Trigonometry, Law of Sines/Cosines, Vectors	December (~15 days)
UNIT 2: Trigonometry Dot Products, Complex Numbers, DeMoivre's Theorem UNIT 3: Analytical Geometry Parametric Functions, Polar Functions	January (~18 days)
UNIT 3: Analytical Geometry Three-dimensional Coordinate System, Vectors in Space, Cross Products UNIT 4: Sequences and Series Arithmetic, Geometric, Binomial Theorem	February (~18 days)
UNIT 5: An Introduction to Calculus Limit Properties	March (~15-20 days)
UNIT 5: An Introduction to Calculus Limit Properties and Applications, Squeeze Theorem, Special Trig Limits	April (~15-20 days)
UNIT 5: An Introduction to Calculus Derivatives Definition, Power Rule, Product/Quotient Rule	May (~18 days)
UNIT 5: An Introduction to Calculus Derivatives - Chain Rule, Exponential, Logarithms, Trig, Inverse Trig Rules	June (~15 days)

Units

Contact the Content Supervisor for unit details.