MOORESTOWN TOWNSHIP PUBLIC SCHOOLS
MOORESTOWN, NEW JERSEY

Moorestown High School
Mathematics

Honors Calculus
Grade 12

Date: February 2020
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Course Description and Fundamental Concepts

This course is for students who have completed Pre-calculus and do not plan on taking the Advanced Placement Calculus Exam. This is a non-trigonometric calculus course with concentration on the reinforcement of algebraic and geometric concepts needed in the applications of calculus. Major topics include: elementary functions, limits, derivatives and definite & indefinite integrals. Business and scientific applications are infused throughout the course. This course does not prepare students for the AP exam. A graphing calculator is required for the course. This course is not for students who have completed Honors Pre-calculus.

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New Jersey Student Learning Standards (NJSLS)

Subject/Content Standards
Include grade appropriate subject/content standards that will be addressed

N-RN The Real Number System

A. Extend the properties of exponents to rational exponents.
   1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define 51/3 to be the cube root of 5 because we want $(5^{1/3})^3 = 5(^{1/3})^3$ to hold, so $(5^{1/3})^3$ must equal 5.
   2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

B. Use properties of rational and irrational numbers.
   3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

N-Q Quantities

A. Reason quantitatively and use units to solve problems.
   1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
   2. Define appropriate quantities for the purpose of descriptive modeling.
   3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

N -CN The Complex Number System

A. Perform arithmetic operations with complex numbers.
   1. Know there is a complex number i such that $i^2 = –1$, and every complex number has the form a + bi with a and b real.
   2. Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
   3. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

C. Use complex numbers in polynomial identities and equations.
   7. Solve quadratic equations with real coefficients that have complex solutions.
   9. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

A-SSE Seeing Structure in Expressions

A. Interpret the structure of expressions
   1. Interpret expressions that represent a quantity in terms of its context.
      a. Interpret parts of an expression, such as terms, factors, and coefficients.
      b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P

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2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

B. Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
    a. Factor a quadratic expression to reveal the zeros of the function it defines.
    b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
    c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.01212t$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

A-APR Arithmetic with Polynomials and Rational Expressions

A. Perform arithmetic operations on polynomials

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

B. Understand the relationship between zeros and factors of polynomials

2. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

C. Use polynomial identities to solve problems

4. Prove polynomial identities and use them to describe numerical relationships. For example, the difference of two squares; the sum and difference of two cubes; the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.
5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

D. Rewrite rational expressions

6. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
7. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

A-CED Creating Equations

A. Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For

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example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.

A-REI Reasoning with Equations and Inequalities

A. Understand solving equations as a process of reasoning and explain the reasoning

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

B. Solve equations and inequalities in one variable

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
4. Solve quadratic equations in one variable.
   a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
   b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

D. Represent and solve equations and inequalities graphically

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
12. Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

F-IF Interpreting Functions

A. Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

B. Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it

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takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

C. Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
    a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
    b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
    c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
    d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
    e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
    a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
    b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

F-BF Building Functions

A. Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.
    a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
    b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
    c. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

B. Build new functions from existing functions

3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using

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technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F-LE Linear and Exponential Models

A. Construct and compare linear and exponential models and solve problems
   1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
      a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
      b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
      c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
   2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
   3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
   4. Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

B. Interpret expressions for functions in terms of the situation they model
   5. Interpret the parameters in a linear or exponential function in terms of a context.

G-GPE Expressing Geometric Properties with Equations

B. Use coordinates to prove simple geometric theorems algebraically
   5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

G-GMD Geometric Measurement and Dimension

A. Explain volume formulas and use them to solve problems

G-MG Modeling with Geometry

A. Apply geometric concepts in modeling situations
   2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot)
   3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios)

Mathematical Practice Standards

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.

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7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

21st-Century Skills and Technology Integration (Standard 8)
List appropriate units below for which strands (A through F) will be addressed

Standard 8.1 (K-12)		**Educational Technology:** *All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.*
Unit Addressed	**Strand Letter**	**Standard Description**
Unit 1, 2, 3, 4	**Strand A**	**Technology Operations and Concepts:** *Students demonstrate a sound understanding of technology concepts, systems, and operations.*
Unit 1, 2, 3, 4	**Strand B**	**Creativity and Innovation:** *Students demonstrate creative thinking, construct knowledge and develop innovative products and process using technology.*
Unit 1, 2, 3, 4	**Strand C**	**Communication and Collaboration:** *Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.*
	Strand D	**Digital Citizenship:** *Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.*
Unit 1, 2, 3, 4	**Strand E**	**Research and Information Fluency:** *Students apply digital tools to gather, evaluate, and use information.*
Unit 1, 2, 3, 4	**Strand F**	**Critical thinking, problem-solving, and decision making:** *Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.*

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Career Ready Practices (Standard 9)
List appropriate units below for which CRPs will be addressed

Unit Addressed	Standard #	Standard Description
Unit 1, 2, 3, 4	**CRP1**	*Act as a responsible and contributing citizen and employee.*
Unit 1, 2, 3, 4	**CRP2**	*Apply appropriate academic and technical skills.*
	CRP3	*Attend to personal health and financial well-being.*
Unit 1, 2, 3, 4	**CRP4**	*Communicate clearly and effectively and with reason.*
Unit 1, 2, 3, 4	**CRP5**	*Consider the environmental, social and economic impacts of decisions.*
Unit 1, 2, 3, 4	**CRP6**	*Demonstrate creativity and innovation.*
	CRP7	*Employ valid and reliable research strategies.*
Unit 1, 2, 3, 4	**CRP8**	*Utilize critical thinking to make sense of problems and persevere in solving them.*
	CRP9	*Model integrity, ethical leadership, and effective management.*
Unit 1, 2, 3, 4	**CRP10**	*Plan education and career paths aligned to personal goals.*
Unit 1, 2, 3, 4	**CRP11**	*Use technology to enhance productivity.*
	CRP12	*Work productively in teams while using cultural global competence*

Interdisciplinary Connections
List any other content standards addressed as well as appropriate units

Visual & Performing Arts Integration (Standard 1)
List appropriate units below for which standards (1.1 through 1.4) may be addressed

Unit Addressed	Standard #	Standard Description
	Standard 1.1	**The Creative Process:** *All students will demonstrate an understanding of the elements and principles that govern the creation of works of art in dance, music, theatre, and/or visual art.*
	Standard 1.2	**History of the Arts and Culture:** *All students will understand the role, development, and influence of the arts throughout history and across cultures.*

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	Standard 1.3	**Performing/Presenting/Producing:** *All students will synthesize those skills, media, methods, and technologies appropriate to creating, performing, and/or presenting works of art in dance, music, theatre, and/or visual art.*
	Standard 1.4	**Aesthetic Responses & Critique Methodologies:** *All students will demonstrate and apply an understanding of arts philosophies, judgment, and analysis to works of art in dance, music, theatre, and/or visual art.*

Other Interdisciplinary Content Standards

List appropriate units below for any other content/standards that may be addressed

Unit Addressed	Content / Standard #	Standard Description
Units 1, 3, 4	**9.1.12.B.2**	Compare strategies for saving and investing and the factors that influence how much should be saved or invested to meet financial goals.
Units 1, 3, 4	**9.1.12.B.8**	Describe and calculate interest and fees that are applied to various forms of spending, debt, and saving.
Units 1, 2, 3, 4	**9.3.12.BM-BIM.3**	Access, evaluate and disseminate information for business decision making.
Units 3, 4	**HS-PS2-1.A**	Forces and Motion Newton's second law accurately predicts changes in the motion of macroscopic objects. (HS-PS2-1) Momentum is defined for a particular frame of reference; it is the mass times the velocity of the object. (HS-PS2-2) If a system interacts with objects outside itself, the total momentum of the system can change; however, any such change is balanced by changes in the momentum of objects outside the system. (HS-PS2-2),(HS-PS2-3)
Units 2, 3, 4	**HS-LS4-6**	Mathematical and computational thinking in 9-12 builds on K-8 experiences and progresses to using algebraic thinking and analysis, a range of linear and nonlinear functions including trigonometric functions, exponentials and logarithms, and computational tools for statistical analysis to analyze, represent, and model data. Simple computational simulations are created and used based on mathematical models of basic assumptions
Unit 1	**HS-PS1-5**	Apply scientific principles and evidence to provide an explanation about the effects of changing the temperature or

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		concentration of the reacting particles on the rate at which a reaction occurs. [Clarification Statement: Emphasis is on student reasoning that focuses on the number and energy of collisions between molecules.] [Assessment Boundary: Assessment is limited to simple reactions in which there are only two reactants; evidence from temperature, concentration, and rate data; and qualitative relationships between rate and temperature.

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Pacing Guide (All Dates are approximate based on the school calendar)

Unit/ Topic	Month (w/Approx number of Teaching Days)
UNIT 1 Precalculus Review	**September** **(~19 days)**
UNIT 2 Functions, Graphs, and Limits	**October** **(~19 days)**
UNIT 3 Differentiation	**November** **(~16 days)**
UNIT 3 Applications of the Derivative	**December** **(~15 days)**
UNIT 3 Applications of the Derivative	**January** **(~18 days)**
UNIT 3 Exponential and Logarithmic Functions	**February** **(~18 days)**
UNIT 3 Exponential and Logarithmic Functions	**March** **(~15-20 days)**
UNIT 4 Integration and Its Applications	**April** **(~15-20 days)**
UNIT 4 Integration and Its Applications	**May** **(~18 days)**
UNIT 4 Techniques of Integration	**June** **(~15 days)**

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Units

Contact the Content Supervisor for unit details.

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