

**MOORESTOWN TOWNSHIP PUBLIC SCHOOLS
MOORESTOWN, NEW JERSEY**

**Moorestown High School
Mathematics**

**Topics in College Mathematics
*Grade 12***

Date: February 2020

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[Course Description and Fundamental Concepts](#)

This course is for students who have passed Algebra II CP but do not feel that they have sufficiently mastered the skills to take College Algebra or Pre-calculus. Major concepts include: linear functions, algebraic analysis of linear functions, quadratic functions, algebraic analysis of quadratic equations, complex numbers, fractional exponents, and exponential functions. This course is not available to those students who had College Algebra, Pre-calculus or Honors Pre-Calculus.

New Jersey Student Learning Standards (NJSL)

Subject/Content Standards

Include grade appropriate subject/content standards that will be addressed

A-APR Arithmetic with Polynomials and Rational Expressions

- A. Perform arithmetic operations on polynomials
 - 1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- B. Understand the relationship between zeros and factors of polynomials
 - 2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
 - 3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- C. Use polynomial identities to solve problems
 - 4. Prove polynomial identities and use them to describe numerical relationships. For example, the difference of two squares; the sum and difference of two cubes; the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.
 - 5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n , where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
- D. Rewrite rational expressions
 - 6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
 - 7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

A-CED Creating Equations

- A. Create equations that describe numbers or relationships
 - 1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
 - 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
 - 3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
 - 4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R .

A-REI Reasoning with Equations and Inequalities

- A. Understand solving equations as a process of reasoning and explain the reasoning
 - 1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
 - 2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- B. Solve equations and inequalities in one variable
 - 3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
 - 4. Solve quadratic equations in one variable.
 - a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
 - b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b .
- C. Solve systems of equations
 - 5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
 - 6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
 - 7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.
- D. Represent and solve equations and inequalities graphically
 - 10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
 - 11. Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
 - 12. Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

F-IF Interpreting Functions

- A. Understand the concept of a function and use function notation
 - 2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- B. Interpret functions that arise in applications in terms of the context
 - 5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
 - 6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

C. Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
 - a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
 - b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
 - c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
 - d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
 - a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
 - b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.

F-BF Building Functions

- A. Build a function that models a relationship between two quantities
 1. Write a function that describes a relationship between two quantities.
 - a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- B. Build new functions from existing functions
 3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F-LE Linear and Exponential Models

- A. Construct and compare linear and exponential models and solve problems
 2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

G-GPE Expressing Geometric Properties with Equations

- A. Translate between the geometric description and the equation for a conic section
 2. Derive the equation of a parabola given a focus and directrix.

S-ID Interpreting Categorical and Quantitative Data

- B. Summarize, represent, and interpret data on two categorical and quantitative variables
 6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

- a. Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.

N -CN The Complex Number System

- A. Perform arithmetic operations with complex numbers.
 1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
 2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
- C. Use complex numbers in polynomial identities and equations.
 7. Solve quadratic equations with real coefficients that have complex solutions.

Mathematical Practice Standards

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

21st-Century Skills and Technology Integration (Standard 8)

List appropriate units below for which strands (A through F) will be addressed

Standard 8.1 (K-12)		Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.
Unit Addressed	Strand Letter	Standard Description
Units 1, 2, 3, 4	Strand A	Technology Operations and Concepts: Students demonstrate a sound understanding of technology concepts, systems, and operations.
Unit 2	Strand B	Creativity and Innovation: Students demonstrate creative thinking, construct knowledge and develop innovative products and process using technology.
Units 1, 3, 4	Strand C	Communication and Collaboration: Students use digital media and environments to communicate and work collaboratively, including at a

		<i>distance, to support individual learning and contribute to the learning of others.</i>
	Strand D	Digital Citizenship: <i>Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.</i>
	Strand E	Research and Information Fluency: <i>Students apply digital tools to gather, evaluate, and use information.</i>
Units 1, 2, 3, 4	Strand F	Critical thinking, problem-solving, and decision making: <i>Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.</i>

Career Ready Practices ([Standard 9](#))

List appropriate units below for which CRPs will be addressed

Unit Addressed	Standard #	Standard Description
	CRP1	<i>Act as a responsible and contributing citizen and employee.</i>
Units 1, 2, 3, 4	CRP2	<i>Apply appropriate academic and technical skills.</i>
Unit 1	CRP3	<i>Attend to personal health and financial well-being.</i>
Unit 2	CRP4	<i>Communicate clearly and effectively and with reason.</i>
	CRP5	<i>Consider the environmental, social and economic impacts of decisions.</i>
Units 1, 2, 3, 4	CRP6	<i>Demonstrate creativity and innovation.</i>
	CRP7	<i>Employ valid and reliable research strategies.</i>
Units 1, 2, 3, 4	CRP8	<i>Utilize critical thinking to make sense of problems and persevere in solving them.</i>
	CRP9	<i>Model integrity, ethical leadership, and effective management.</i>
	CRP10	<i>Plan education and career paths aligned to personal goals.</i>
Units 1, 2, 3, 4	CRP11	<i>Use technology to enhance productivity.</i>

	CRP12	<i>Work productively in teams while using cultural global competence</i>
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Interdisciplinary Connections

List any other content standards addressed as well as appropriate units

Visual & Performing Arts Integration ([Standard 1](#))

List appropriate units below for which standards (1.1 through 1.4) may be addressed

Unit Addressed	Standard #	Standard Description
	Standard 1.1	The Creative Process: All students will demonstrate an understanding of the elements and principles that govern the creation of works of art in dance, music, theatre, and/or visual art.
	Standard 1.2	History of the Arts and Culture: All students will understand the role, development, and influence of the arts throughout history and across cultures.
Units 1, 2, 3, 4	Standard 1.3	Performing/Presenting/Producing: All students will synthesize those skills, media, methods, and technologies appropriate to creating, performing, and/or presenting works of art in dance, music, theatre, and/or visual art.
	Standard 1.4	Aesthetic Responses & Critique Methodologies: All students will demonstrate and apply an understanding of arts philosophies, judgment, and analysis to works of art in dance, music, theatre, and/or visual art.

Other Interdisciplinary Content Standards

List appropriate units below for any other content/standards that may be addressed

Unit Addressed	Content / Standard #	Standard Description
Unit 1	Financial Literacy / 9.1.12.A.8	Analyze different forms of currency and how currency is used to exchange goods and services.
Unit 2	Science / ESS4.1	Use mathematical or computational representations to predict the motion of orbiting objects in the solar system
Unit 3	Career and Technical Education / 9.3.12.AG-ENV.3	Develop proposed solutions to environmental issues, problems and applications using scientific principles of meteorology, soil science, hydrology, microbiology, chemistry and ecology.

Unit 4	Career and Technical Education / 9.3.12.AC-DES.6	Apply the techniques and skills of modern drafting, design, engineering and construction to projects.
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Pacing Guide (All Dates are approximate based on the school calendar)

Unit/ Topic	Month (w/Approx number of Teaching Days)
<p style="text-align: center;">UNIT 1 Fundamentals of Algebra, Linear Equations and Inequalities</p>	<p style="text-align: center;">September (~19 days)</p>
<p style="text-align: center;">UNIT 1 Linear Equations and Inequalities</p> <p style="text-align: center;">UNIT 2 Graphs and Functions</p>	<p style="text-align: center;">October (~19 days)</p>
<p style="text-align: center;">UNIT 2 Graphs and Functions</p>	<p style="text-align: center;">November (~16 days)</p>
<p style="text-align: center;">UNIT 2 Graphs and Functions</p>	<p style="text-align: center;">December (~15 days)</p>
<p style="text-align: center;">UNIT 2 Graphs and Functions</p>	<p style="text-align: center;">January (~18 days)</p>
<p style="text-align: center;">UNIT 3 Polynomials and Factoring</p>	<p style="text-align: center;">February (~18 days)</p>
<p style="text-align: center;">UNIT 3 Polynomials and Factoring</p>	<p style="text-align: center;">March (~15-20 days)</p>
<p style="text-align: center;">UNIT 3 Rational Expressions, Equations, and Functions</p>	<p style="text-align: center;">April (~15-20 days)</p>
<p style="text-align: center;">UNIT 3 Rational Expressions, Equations and Functions</p> <p style="text-align: center;">UNIT 4 Radicals and Complex Numbers</p>	<p style="text-align: center;">May (~18 days)</p>
<p style="text-align: center;">UNIT 4 Radical and Complex Numbers</p>	<p style="text-align: center;">June (~15 days)</p>

[Units](#)

Contact the Content Supervisor for unit details.