

**MOORESTOWN TOWNSHIP PUBLIC SCHOOLS
MOORESTOWN, NEW JERSEY**

**Moorestown High School
Mathematics**

**AP Calculus BC
*Grades 11-12***

Date: February 2020
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[Course Description and Fundamental Concepts](#)

This course is offered to students who have completed Honors Pre-calculus. It prepares students to take the "Calculus BC" AP Exam. The course is ambitious, the pace is fast, and students are expected to share responsibility for their learning. Major topics include: limits, differential and integral calculus, sequences and series, elementary differential equations, and hyperbolic functions. Graphing calculators are an integral part of the curriculum. A graphing calculator is required.

New Jersey Student Learning Standards (NJSLS)

Subject/Content Standards

Include grade appropriate subject/content standards that will be addressed

N-RN The Real Number System

- A. Extend the properties of exponents to rational exponents.
 - 1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3) \cdot 3} = 5^1 = 5$ to hold, so $(5^{1/3})^3$ must equal 5.
 - 2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

N-Q Quantities

- A. Reason quantitatively and use units to solve problems.
 - 1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
 - 2. Define appropriate quantities for the purpose of descriptive modeling.
 - 3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

N-CN The Complex Number System

- C. Use complex numbers in polynomial identities and equations.
 - 7. Solve quadratic equations with real coefficients that have complex solutions.
 - 9. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

A-APR Arithmetic with Polynomials and Rational Expressions

- A. Perform arithmetic operations on polynomials
 - 1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- B. Understand the relationship between zeros and factors of polynomials
 - 3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- C. Rewrite rational expressions
 - 4. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
 - 5. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

D. Rewrite rational expressions

6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
7. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

A-CED Creating Equations

- A. Create equations that describe numbers or relationships
 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
 4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R .

A-REI Reasoning with Equations and Inequalities

- A. Understand solving equations as a process of reasoning and explain the reasoning
 2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- B. Solve equations and inequalities in one variable
 3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- D. Represent and solve equations and inequalities graphically
 10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

F-IF Interpreting Functions

- A. Understand the concept of a function and use function notation
 1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
 2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- B. Interpret functions that arise in applications in terms of the context
 4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
 5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
 6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- C. Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
 - a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
 - b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
 - c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
 - d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
 - e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
 - a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
 - b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.

F-BF Building Functions

B. Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F-TF Trigonometric Functions

A. Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for πx , $\pi + x$, and $2\pi - x$ in terms of their values for x , where x is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

C. Prove and apply trigonometric identities

8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
9. Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

G-GPE Expressing Geometric Properties with Equations

B. Use coordinates to prove simple geometric theorems algebraically

5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

G-GMD Geometric Measurement and Dimension

- A. Explain volume formulas and use them to solve problems

G-MG Modeling with Geometry

- A. Apply geometric concepts in modeling situations
 2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot)
 3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios)

Mathematical Practice Standards

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

21st-Century Skills and Technology Integration (Standard 8)

List appropriate units below for which strands (A through F) will be addressed

Standard 8.1 (K-12)		Educational Technology: <i>All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.</i>
Unit Addressed	Strand Letter	Standard Description
Units 1, 2, 3, 4	Strand A	Technology Operations and Concepts: <i>Students demonstrate a sound understanding of technology concepts, systems, and operations.</i>
Units 1, 2, 3, 4	Strand B	Creativity and Innovation: <i>Students demonstrate creative thinking, construct knowledge and develop innovative products and process using technology.</i>
Units 1, 2, 3, 4	Strand C	Communication and Collaboration: <i>Students use digital media and environments to communicate and work collaboratively, including at a</i>

		<i>distance, to support individual learning and contribute to the learning of others.</i>
Units 1, 2, 3, 4	Strand D	Digital Citizenship: <i>Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.</i>
Units 1, 2, 3, 4	Strand E	Research and Information Fluency: <i>Students apply digital tools to gather, evaluate, and use information.</i>
Units 1, 2, 3, 4	Strand F	Critical thinking, problem-solving, and decision making: <i>Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.</i>

Career Ready Practices ([Standard 9](#))

List appropriate units below for which CRPs will be addressed

Unit Addressed	Standard #	Standard Description
	CRP1	<i>Act as a responsible and contributing citizen and employee.</i>
Units 1, 2, 3, 4	CRP2	<i>Apply appropriate academic and technical skills.</i>
	CRP3	<i>Attend to personal health and financial well-being.</i>
Units 1, 2, 3, 4	CRP4	<i>Communicate clearly and effectively and with reason.</i>
	CRP5	<i>Consider the environmental, social and economic impacts of decisions.</i>
Units 1, 2, 3, 4	CRP6	<i>Demonstrate creativity and innovation.</i>
Units 1, 2, 3, 4	CRP7	<i>Employ valid and reliable research strategies.</i>
Units 1, 2, 3, 4	CRP8	<i>Utilize critical thinking to make sense of problems and persevere in solving them.</i>
	CRP9	<i>Model integrity, ethical leadership, and effective management.</i>
Units 1, 2, 3, 4	CRP10	<i>Plan education and career paths aligned to personal goals.</i>
Units 1, 2, 3, 4	CRP11	<i>Use technology to enhance productivity.</i>
	CRP12	<i>Work productively in teams while using cultural global competence</i>

Interdisciplinary Connections

List any other content standards addressed as well as appropriate units

Visual & Performing Arts Integration ([Standard 1](#))

List appropriate units below for which standards (1.1 through 1.4) may be addressed

Unit Addressed	Standard #	Standard Description
	Standard 1.1	The Creative Process: All students will demonstrate an understanding of the elements and principles that govern the creation of works of art in dance, music, theatre, and/or visual art.
Units 3, 4	Standard 1.2	History of the Arts and Culture: All students will understand the role, development, and influence of the arts throughout history and across cultures.
Units 3, 4	Standard 1.3	Performing/Presenting/Producing: All students will synthesize those skills, media, methods, and technologies appropriate to creating, performing, and/or presenting works of art in dance, music, theatre, and/or visual art.
	Standard 1.4	Aesthetic Responses & Critique Methodologies: All students will demonstrate and apply an understanding of arts philosophies, judgment, and analysis to works of art in dance, music, theatre, and/or visual art.

Other Interdisciplinary Content Standards

List appropriate units below for any other content/standards that may be addressed

Unit Addressed	Content / Standard #	Standard Description
Units 1, 2, 3, 4	HS-PS2-1	Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration.
Unit 2	HS-PS3-4	Plan and conduct an investigation to provide evidence that the transfer of thermal energy when two components of different temperature are combined within a closed system results in a more uniform energy distribution among the components in the system (second law of thermodynamics).
Unit 1	HS-PS1-8	Develop models to illustrate the changes in the composition of the nucleus of the atom and the energy released during the processes of fission, fusion, and radioactive decay.

Unit 3	HS-PS4-1	Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media.
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Pacing Guide (All Dates are approximate based on the school calendar)

Unit/ Topic	Month (w/Approx number of Teaching Days)
<p align="center">UNIT 1: Differentiation Limits and Their Properties Basic Differentiation Applications of Differentiation</p>	<p align="center">September (~19 days)</p>
<p align="center">UNIT 1: Differentiation Applications of Differentiation Logarithmic, Exponential, and Other Transcendental Functions Differential Equations</p>	<p align="center">October (~19 days)</p>
<p align="center">UNIT 2: Integration Basic Integration Logarithmic, Exponential, and Other Transcendental Functions</p>	<p align="center">November (~16 days)</p>
<p align="center">UNIT 2: Integration Applications of Integration</p>	<p align="center">December (~15 days)</p>
<p align="center">UNIT 2: Integration Integration Techniques & Improper Integrals</p>	<p align="center">January (~18 days)</p>
<p align="center">UNIT 3: Infinite Series, Polar and Parametric Equations Infinite Series (Sequences and Series)</p>	<p align="center">February (~18 days)</p>
<p align="center">UNIT 3: Infinite Series, Polar and Parametric Equations Infinite Series (Sequences and Series) Vector and the Geometry of Space Vector-Valued Functions</p>	<p align="center">March (~15-20 days)</p>
<p align="center">UNIT 3: Infinite Series, Polar and Parametric Equations Conics, Parametric Equations, and Polar Coordinates/Equations</p>	<p align="center">April (~15-20 days)</p>
<p align="center">UNIT 1, UNIT 2, UNIT 3 AP Test Review, AP Test UNIT 4 Topics Beyond BC</p>	<p align="center">May (~18 days)</p>
<p align="center">UNIT 4 Topics Beyond BC</p>	<p align="center">June (~15 days)</p>

[Units](#)

Contact the Content Supervisor for unit details.